A possible interpretation of the steady state Poynting vector field in a physical system

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“. . . metals are good conductors of current but not good conductors of energy. Metals conduct current but the space around them conducts energy and the best conductor of energy is a vacuum!” A. Sommerfeld, Electrodynamics, Academic Press, New York, 1952.

“The energy transfer takes place, in the neighborhood of the wire, almost parallel to it, with a small tangent to the wire. . . Prof. Poynting [1885], on the other hand, proposed a different view, representing the energy transfer almost perpendicular to the wire, i.e., with a small deviation from the vertical.
I think that this difference of one quadrant arises only from what appears to be a misconception on his part as to what the nature of the electric field is like in the neighborhood of a wire carrying a current. The lines of force of the electric field are almost perpendicular to the wire.” P.J. Nahin, O. J. Heaviside, The Life, Work, and Times of an Electrical Genius of the Victorian Age, The Johns Hopkins University Press, Baltimore, 2002. The reference to Poynting is the following: Poynting, J.H., On the Connection Between Electric Current and Electric and Magnetic Inductions in the Surrounding Field, Phil. Trans., 176, 277-306, 1885.

“The electrons inside the conductor are driven by an electric field that has been created by electrical charges that are very far away and the energy that these electrons acquire is finally transformed into heat.” R.P. Feynman, R.B Leighton and M. Sands, The Feynman Lectures on Physics, volume 2, Addison-Wesley, Reading, 1965.

Abstract: **A possible interpretation of the steady state Poynting vector field when both, the electric and the magnetic field are interrelated and stem from the steady charge distribution and charge flow in a physical system is suggested from the analysis of several examples.**

This Tech Note gives an interpretation of the steady state Poynting vector field when both, the electric and the magnetic field are interrelated and stem from the steady charge distribution and charge flow in a given physical system. This approach makes it possible to overcome the objections raised by people who maintain that Poynting vector fields have meaning as flows of electromagnetic energy density only when related with electromagnetic waves.

It could be of certain interest to some teachers of physics or electrical engineering, as well as to students of physics or engeneering sciences.

As Georg Joos once obseved in his book about theoretical physics, it seems that the Poynting Vector Field **S**, with its meaning related with both energy and momentum, **if suitably constructed**, can be applied in all cases, apparently never coming into conflict neither with experience nor conservation laws (G.Joos and I.Freeman, “Theoretical Physics”, 3ª edición, Dover, N.Y., 1986, p 332).

However, a Poynting vector field constructed with an arbitrary combination of electric and magnetic fields apparently has no physical meaning, as was stressed, for example, by

Max Abraham and Richard Becker in their "Classical Theory of Electricity and Magnetism", Blackie & Son, Glasgow, 2nd Ed. , 1950, pp 151-152.

These authors say there that it is only when it is taken over a closed surface that the integral of the normal component of the Poynting vector **S**=**E**×**H** has the physical significance of a flow of energy from the region enclosed by the surface.

As an example, they give the case of an arbitrary electrostatic field crossed by an arbitrary magnetic field, such that **S** has zero divergence and therefore "can have no effect on the energy balance". [1]

To take a closer look to the problem, let us consider first an idealized electric circuit whose components are a battery, two zero resistance wires and a resistive charge, connected as shown in the following Figure 1.

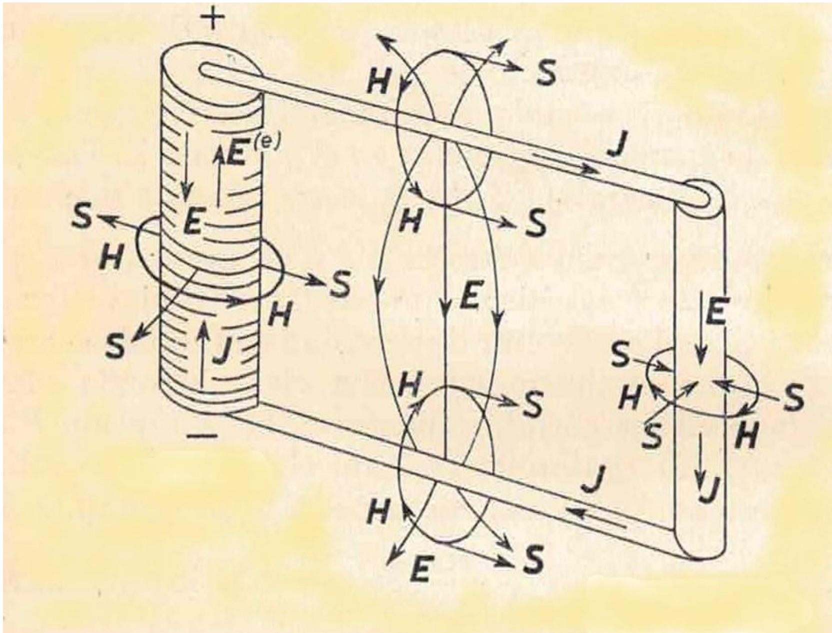

Figure 1 Example given by Werner Döring's Einführung in die Theoretische Physik, Vol. II, Das Elektromagnetische Feld.

Let us summarize the stationary process that appears schematized in the figure above.
The battery appears in the left part of the figure, and the resistive charge (electric resistance) appears in the right part. It is a flat electrical circuit, as suggested in the figure. A continuous electric current J flows through the system as shown in the figure. As usual, the electric field is represented by E, the magnetic field by H and the Poynting field by S.

In this example, the electric field and the magnetic field in steady state are a consequence of the current flow in the system (the whole battery-zero resistance wires-resistive charge).

[1] Taking this position to an extreme, some people think that we might not assign physical meaning to the local values of a Poynting field.

Let us suppose that the battery is a straight and spatially uniform circular cylinder, so both Planck's field $E^{(e)}$ and electric field can be considered as constant in is interior and of opposite sense.

As shown, J flows inside the battery from the negative terminal towards the positive terminal and flows inside the electric resistance from the positive pole to the negative pole. The electrical resistance is assumed to be a straight circular cylinder of constant resistivity. The lines of H surround the electric current in the battery, in the zero resistance wires and in the electrical resistance, as shown in the figure. The lines of the electric field in the plane of the circuit go from the positive wire without resistance to the negative wire without resistance, as can be seen in the figure. Both wires are assumed to be straight circular cylinders. The electric field can be considered as constant inside the battery, while the magnitude of the magnetic field increases in proportion to the radial distance to the battery axis. So, the Poynting vector will be orthogonal to the battery's axis, will be directed from inside towards outside, and **inside the battery** the magnitude of S will grow proportionally to the radial distance to the axis of the cylinder: the energy flow begins near the axis and increases with the radial distance until the surface of the battery is attained.

Along the wires without resistance the Poynting vector will be parallel to the axis of each of those wires. In both wires S will be directed from the battery towards the electric resistance, as suggested in the figure. In the electric resistance the Poynting vector has the radial direction again, but now it is directed towards the axis of the cylinder, with a magnitude that decreases proportionally to the decrease in the distance to the axis. So, here the flow of energy enters and is delivered to the charge carriers, and these carriers delivers energy to the obstacles that oppose their movement.

Now, let us eliminate the zero resistance wires and substitute the straight cylindrical resistive charge by two twisted resistive wire of uniform conductivity and radius, connected in series like the ones shown in Figure 2.

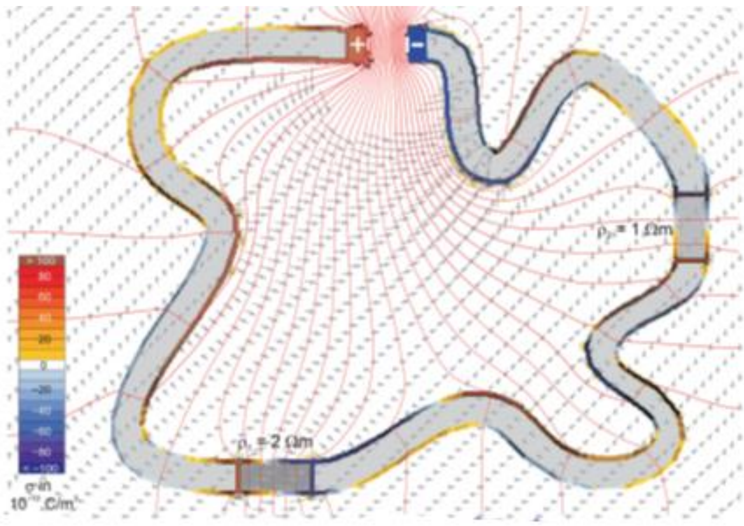

Figure 2 A sketch of the electric field, equipotential lines (red curves), and surface charges for a series connection of two resistors with arbitrarily twisted pieces of hookup wire. Adapted from Rainer Müller, A semiquantitative treatment of surface charges in DC circuits, American Journal of Physics, **80** (9): 782-788, 2012.

In the figure above the density of surface charges is shaded from red to violet, representing most positive to most negative, respectively. The scale gives the surface charge density in 10^{-12} C/m^2.
As suggested in Figure 2 above, the conductors of a current-carrying circuit must have nonuniform surface charge densities. This spatial distribution of charge density assures the confined flow of current inside the conductors, maintains the electric potential around the circuit as well as the electric field in the space outside the conductors. The spatial distribution of surface charges in the conductors and the associated electric fields inside and outside the conductors depend on the geometry and the conductivities of the different parts of the circuit and the emf sources.

The electric field in a cross-section located in a curved segment of a non-zero resistance wires is in general not uniform (varies from one point to another one in the same cross-section) although in steady state (DC) its radial component is zero (equipotential cross-sections, as shown in Figure 2). As consequence, the electric current densities in non-zero resistance wires are in general not uniform in the cross-sections of these steady state conductors. This complex steady spatial distribution of electric current densities generates a correspondingly complex steady spatial distribution of magnetic fields inside and outside the conductors.

But both vector fields, the electric and the magnetic ones are closely interrelated and stem from the steady charge distribution and flow in the considered system. The same emf produces different patterns of electric and magnetic fields, surface densities of electric charges and charge flows in systems of conductors with different geometries and electric conductivities.

In the book by Oleg Jefimenko, Electricity and Magnetism, Plenum, New York, 1966, several examples of the above-mentioned interrelations can be found.

Even if you could shield the electric field between the wires that appear in the examples, with no influence on the delivered power to the load, then you couldn´t maintain the same distribution of surface charges and current densities (and local electric energy dissipation) in the cross sections.
An external electric field near the surface of twisted conductors would appear related to the distribution of surfaces charges that maintain the charge flow within the boundaries, and an external magnetic field due to the currents inside. And then, a Poynting field could be constructed.
An example of this role of Poynting vector field **S** in a steady state system can be found in the article due to J. D. Jackson: Surface charges on circuit wires and resistors play three roles, American Journal of Physics, **64**:855-870, 1996. Here Jackson gives a relatively simple mathematical model of circuits consisting of a resistor and a battery connected by wires and other conductors, in geometries that allow the construction of closed form analytic solutions by means of Fourier–Bessel series.

Besides the above-mentioned examples of meaningful interpretations of **S** for steady fields in suitable defined systems, there are suggestions that the meaning of **S** can be extended to static fields under appropriate circumstances, like the ones studied in Pugh and Pugh "Physical significance of the Poynting Vector in static fields" American Journal of Physics 35, 153 (1967); https://doi.org/10.1119/1.1973915